time, A; N_1, a proportionality factor, W/° C; N_2, a proportionality factor, A/°C; $\Delta T_{e,max}$, maximum temperature drop from gas of charge carriers to crystal lattice, °C; $\Delta T_{L,max}$, maximum temperature drop from crystal lattice to ambient medium, °C; ΔT_{max}, maximum temperature drop from thermistor to ambient medium, °C; ΔT_{crB} and ΔT_{crC}, temperature drops corresponding to points B and C, respectively, °C; $d\Delta T_{crB}/dt$ and $d\Delta T_{crC}/dt$, numerical values of the derivative at points B and C, respectively, °C/sec; δ, a constant [7]; M, Lipschitz constant; t, time, sec; H, dissipation coefficient of the thermistor, W/°C; and C_V, volumetric heat capacity of the thermistor, W · sec/°C.

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EFFECT OF THE GEOMETRY OF THE MOVING WALL ON THE STRUCTURE OF THE FLOW IN CONFINED FLOW IN A SLIT GAP

A. A. Volkov and L. V. Poluyanov

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An analytical solution of the problem is presented with an estimate of the effect of the wavy oscillating wall on the flow characteristics of a viscous liquid in a slit gap.

When analyzing the heat- and mass-exchange characteristics between a flow of liquid and a solid surface one must bear in mind that the geometry of the channel walls has a considerable influence on the structure of the flow. Experimental methods are widely used to solve this problem because of the mathematical difficulties. In [1] an estimate is made of the effect of the microgeometry of the surface on the structure of the flow based on a solution of the problem of Couette flow with a fixed wavy wall.

We will consider the more general nonstationary case when the wavy wall performs harmonic oscillations, the flow is confined, and the gap between the walls h is fairly small compared with the characteristic length of the channel. The Reynolds number is assumed to be very much less than 1.

The law of motion of the lower wall and the equations of the upper and lower walls can be written in the form

$$\begin{gathered} x_t = x_0 + a\sin\omega t; \\ y_l(x,\ t) = \bar{e}\sin k(x - a\sin\omega t); \\ y_u = h = \text{const}, \end{gathered} \tag{1}$$

where a and $\bar{e}$ are the amplitudes of oscillation of the wall and the wavy surface; k, wave number; and ω,

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frequency. We will introduce the following dimensionless parameters: $\alpha = hk$; $\alpha_1 = ak$; $\varepsilon = \bar{e}/h$; $\beta = k\Delta p/\rho a\omega^2$; $\gamma = \nu k^2/\omega$. In this case Stokes' equations, the continuity equations, and the boundary condition take the form

$$\frac{\partial u}{\partial t} = -\beta\frac{\partial p}{\partial x} + \gamma\left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{\alpha^2}\frac{\partial^2 u}{\partial y^2}\right);$$
$$\frac{\partial v}{\partial t} = -\frac{\beta}{\alpha^2}\frac{\partial p}{\partial y} + \gamma\left(\frac{\partial^2 v}{\partial x^2} + \frac{1}{\alpha^2}\frac{\partial^2 v}{\partial y^2}\right);$$
$$\frac{\partial u}{\partial x} + \varepsilon\frac{\partial v}{\partial y} = 0; \tag{2}$$
$$y = \varepsilon\sin(x - \alpha_1\sin t);\ v = 0;\ u = \mathrm{cost};$$
$$y = 1;\ v = u = 0.$$

Assuming that $\varepsilon \gg 1$, and that β, γ are arbitrary quantities on the order of O(1), the solution will be sought in the form of an expansion in series with respect to the small parameter

$$u = u_0 + \varepsilon u_1 + \ldots,$$
$$v = v_0 + \varepsilon v_1 + \ldots, \tag{3}$$
$$p = p_0 + \varepsilon p_1 + \ldots$$

We will confine ourselves to considering the linear approximation (oscillations of a smooth wall and waviness effects which appear on this "background"). Obviously, for the zeroth approximation $\varepsilon = 0$ and $v = 0$, and the mathematical problem reduces to solving an equation of the thermal conduction type with boundary conditions of the first kind. The velocity profile for the zeroth approximation (in dimensional quantities) has the form

$$u_0 = \frac{\Delta p}{2\mu k h^2} y(y - h) + a\omega f_{0,r}\left(\frac{y}{h}\right)\cos\omega t - a\omega f_{0,i}\left(\frac{y}{h}\right)\sin\omega t, \tag{4}$$

where

$$f_0 = \mathrm{sh}\,\lambda(y - 1)/\mathrm{sh}\lambda;\ \lambda = \frac{1+i}{\sqrt{2\gamma}}. \tag{5}$$

It is easy to see that in this case the longitudinal velocity is the sum of the Poiseuille and oscillating velocity profile. The solution corresponds in its physical meaning to the similar well-known parallel motion of a viscous liquid in a gap between upper and lower walls moving with constant velocity [3].

Consider the effects of the first approximation. The first approximation for u and p will be found in the form of a system of equations (2) in which we put $u = u_1$ and $v = v_0$. The boundary conditions on the fixed wall are

$$y = 1;\ v_0 = u_1 = 0, \tag{6}$$

and for the moving wall, from [4] we have

$$u(y_m) = u(0) + \left.\frac{\partial u}{\partial y}\right|_0 y_m + \frac{1}{2}\left.\frac{\partial^2 u}{\partial y^2}\right|_0 y_m^2 + \ldots,$$
$$v(y_m) = v(0) + \left.\frac{\partial v}{\partial y}\right|_0 y_m + \frac{1}{2}\left.\frac{\partial^2 v}{\partial y^2}\right|_0 y_m^2 + \ldots \tag{7}$$

Substituting series (3) for u and v and taking the boundary conditions into account we finally obtain

$$y = 0;\ v_0 = 0;\ u_1 = \left[\frac{\beta}{2\gamma} - f'_{0,r}(0)\cos t + f'_{0,i}(0)\sin t\right]\sin(x - \alpha_1\sin t), \tag{8}$$

where $f'_{0,r}(0) = -\mathrm{Real}\,\lambda\coth\lambda; f'_{0,i}(0) = -\mathrm{Im}\,\lambda\coth\lambda$.

Since all the required functions are periodic in x and t, the solution will be sought in the form of double complex Fourier series [5]

$$u_1 = \sum_{\kappa,l}\sigma_{\kappa l}(y)\,e^{i(\kappa x + lt)};$$

$$v_0 = \sum_{m,n} \tau_{mn}(y)\, e^{i(mx+nt)};$$
$$p_1 = \sum_{r,s} \pi_{r,s}(y)\, e^{i(rx+st)}, \qquad (9)$$

where the summation is carried out over the integer numbers from $-\infty$ to $+\infty$.

Substituting (9) into the initial system of equations and equating the coefficients with the same exponents, we obtain a system of equations for determining σ, τ, π:

$$\sigma''_{\kappa l} - \left(\kappa^2 + \frac{il}{\gamma}\right)\sigma_{\kappa l} = \frac{i\kappa\beta}{\gamma}\pi_{\kappa l};$$
$$\tau''_{\kappa l} - \left(\kappa^2 + \frac{il}{\gamma}\right)\tau_{\kappa l} = \frac{\beta}{\gamma}\pi'_{\kappa l}; \qquad (10)$$
$$i\kappa\sigma_{\kappa l} + \tau'_{\kappa l} = 0.$$

The boundary conditions on the fixed wall are

$$y = 1;\ \sigma_{\kappa l} = 0;\ \tau_{\kappa l} = 0. \qquad (11)$$

On the movable wall with $y = 0$, $\tau = 0$, and σ, taking (9) into account, expressed in terms of its original [5], has the final form (we omit the fairly lengthy calculations)

$$\sigma_{\kappa l}(0) = \frac{i\beta}{\gamma}[\delta_{-1\kappa} - (-1)^e\delta_{1\kappa}]J_l(\alpha_1) - \frac{i}{4}[\delta_{-1\kappa} + (-1)^e\delta_{1\kappa}][f'_0(0)J_{l-1}(\alpha_1) + f'(0)J_{l+1}(\alpha_1)], \qquad (12)$$

where δ_k is the Kronecker delta; $J_l(\alpha)$, Bessel function of the l-th order of argument α; and the prime on the function f denotes the complex conjugate.

As follows from the boundary conditions, the components of the solution with $k = \pm 1$ differ from zero while the series in l is infinite.

Solving (10)-(12) we have

$$\tau = \begin{cases} c_1e^y + c_2y^{-y} + c_3e^{\omega y} + c_4e^{-\omega t}; \\ (c_1^{(0)} + c_2^{(0)}y)\, e^y + (c_3^{(0)} + c_4^{(0)}y)\, e^{-y}, \end{cases} \qquad (13)$$

where

$$\omega = \sqrt{1 + \frac{il}{\gamma}};\ l \neq 0,\ l = 0.$$

We can now find $\sigma_{\pm1,l}$ and $\pi_{\pm1,l}$: for $l \neq 0$

$$\sigma_{\pm1,l}(y) = \pm i(c_1^{\pm}e^y - c_2^{\pm}e^{-y} + \omega c_3^{\pm}y^{\omega y}) - \omega c_4 e^{-\omega y};$$
$$\pi_{\pm1,l}(y) = -\frac{il}{\beta}(c_1^{\pm}e^y - c_2^{\pm}e^{-y}); \qquad (14)$$

and for $l = 0$

$$\sigma_{\pm1,0}(y) = \pm i[(c_1^{(0)\pm} + c_2^{(0)\pm} + c_2^{(0)}y)\, e^y + (c_4^{(0)\pm} - c_5^{(0)\pm} - c_4^{(0)\pm}y)\, e^y];$$
$$\pi_{\pm1,0}(y) = \frac{2\gamma}{\beta}(c_2^{(0)\pm}e^y + c_4^{(0)\pm}e^y). \qquad (15)$$

Using the equations obtained, in which all the quantities are known, while the constants can be determined from the boundary conditions, the solution of the equations of the linear approximation can be found in the form

$$u_1 = \sum_l [\sigma_{1,l}(y)\, e^{ix} + \sigma_{-1,l}(y)\, e^{-ix}]\, e^{ilt};$$
$$v_0 = \sum_l [\tau_{1,l}(y)\, e^{ix} + \tau_{-1,l}(y)\, e^{-ix}]\, e^{ilt}; \qquad (16)$$
$$p_1 = \sum_l [\pi_{1,l}(y)\, e^{ix} + \pi_{-1,l}(y)\, e^{-ix}]\, e^{ilt}.$$

As a specific example we will consider the limiting case of hf and lf oscillations of the wavy wall, i.e.,

$$\gamma = \frac{\nu k}{\omega} \ll 1; \ \beta = \frac{k\Delta p}{\rho a \omega^2} \ll 1; \ \gamma \sim \beta.$$

Under these conditions the velocity profile in the zeroth approximation (see Eq. (4)) has the form

$$u_0 = \frac{k\Delta p}{2\mu} y(y-h) + a\omega e^{-\frac{y}{h\sqrt{2\gamma}}} \cos\left(\frac{y}{h\sqrt{2\gamma}} - \omega t\right). \tag{17}$$

The amplitude of the oscillating flow, superimposed on the velocity profile of the Poiseuille flow, decays exponentially with distance from the oscillating wall with a depth of penetration of the order of $h\sqrt{2\gamma}$ and an oscillation length of $2\pi h\sqrt{2\gamma}$. Consequently, along the velocity profile of the confined flow there travels a wave with phase velocity

$$V_\varphi = \sqrt{2\nu\omega}. \tag{18}$$

Consider the effects of the first approximation, which will interact with those described above.

Since $f_0 = \exp(-\lambda y)$, we have $f_0' = -\exp(i\pi/4)\sqrt{\gamma}$, and we can simplify the expression for $c_{\pm 1,l}(\beta, \gamma, \alpha_1)$, assuming also that $\alpha_1 \ll 1$.

Since the solution has the same order of magnitude as the boundary conditions (when all the remaining boundary conditions are equal to zero), then, confining ourselves to the principal term of the expansion in α_1 and γ, we need retain only those terms of the series in l which correspond to $l = \pm 1$. We will also neglect terms of the order of $O(\gamma^{-1/2})$ compared with terms of the order of $O(1)$ and we finally obtain

$$\begin{aligned} u_1 &= \sigma_{1,1}e^{i(x+t)} + \sigma_{1,-1}e^{i(x-t)} + \sigma_{-1,1}e^{-i(x-t)} + \sigma_{-1,-1}e^{-i(x+t)}; \\ v_0 &= \tau_{1,1}e^{i(x+t)} + \tau_{1,-1}e^{i(x-t)} + \tau_{-1,1}e^{-i(x-t)} + \tau_{-1,-1}e^{-i(x+t)}; \\ p_1 &= \pi_{1,1}e^{i(x+t)} + \pi_{1,-1}e^{i(x-t)} + \pi_{-1,1}e^{-i(x-t)} + \pi_{-1,-1}e^{-i(x+t)}. \end{aligned} \tag{19}$$

Omitting the long and fairly tedious calculations, we will give the results obtained for σ, π, τ

$$\begin{aligned} \sigma_{\pm 1,0} &= \pm i \frac{\beta}{\gamma}[(0.143 - 0.182y)e^{y} + (-0.39 + 0.715y)e^{-y}]; \\ \pi_{\pm 1,0} &= -1.43(0.254e^{y} + e^{-y}); \\ \tau_{\pm 1,0} &= \frac{\beta}{\gamma}[(0.325 - 0.182y)e^{y} - (0.325 + 0.715y)e^{-y}]. \end{aligned} \tag{20}$$

The final solution has the form

$$\begin{aligned} u_1 &= \text{Real}\, F_1(y)e^{it}\sin x + F_0(y)\sin x + \ldots, \\ v_0 &= \text{Real}\, G_1(y)e^{it}\cos x + G_0(y)\cos x + \ldots, \\ p_1 &= \text{Real}\, H_1(y)e^{it}\cos x + H_0(y)\cos x + \ldots, \end{aligned} \tag{21}$$

where

$$\begin{aligned} F_0 &= -2\frac{\beta}{\gamma}[(0.143 - 0.182y)e^{y} + (-0.39 + 0.715y)e^{-y}]; \\ G_0 &\doteq 2\frac{\beta}{\gamma}[(0.325 - 0.182y)e^{y} - (0.325 + 0.715y)e^{-y}]; \\ H_0 &= -2.86(0.254e^{y} + e^{-y}); \\ F_1 &= \frac{1}{\sqrt{\gamma}}e^{-\frac{y}{\sqrt{2\gamma}}}\left[\cos\left(\frac{y}{\sqrt{2\gamma}} - \frac{\pi}{4}\right) - i\sin\left(\frac{y}{\sqrt{2\gamma}} - \frac{\pi}{4}\right)\right]; \\ G_1 &= -\frac{\text{sh}(1-y)}{\text{sh}\,1} + e^{-\frac{y}{\sqrt{2\gamma}}}\left(\cos\frac{y}{\sqrt{2\gamma}} - i\sin\frac{y}{\sqrt{2\gamma}}\right); \\ H_1 &= -\frac{i\,\text{ch}(1-y)}{\beta\,\text{sh}\,1}. \end{aligned} \tag{22}$$

The expressions obtained have an asymptotic character ($\gamma \to 0$).

The qualitative nature of the flow pattern for solutions of the first approximation are shown in the figure. Close to the wavy moving wall there is a circulation-periodic (with respect to x) flow. These perturbations

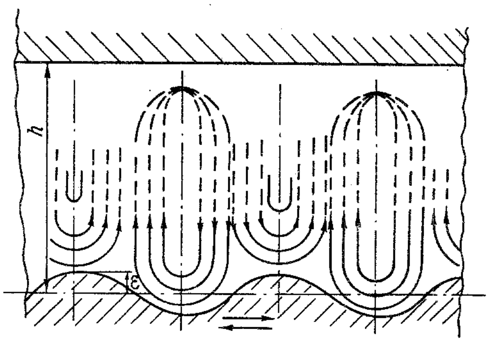

Fig. 1. Qualitative nature of the flow pattern of a viscous liquid in a plane channel with a wavy wall.

decay on the fixed wall. The flow described by terms with indices 1 (Eq. (21)) represent a sequence of coupled "oscillating" circulations with a direction of the perturbations of the velocity on the surface in the regions $\sin x < 0$, opposite to the motion of the wall.

The flow described by terms with indices 0 is stationary circulation-periodic (with respect to x) flow. Unlike the previous motion, motion here also exists inside the circulation zone with velocities which are $O(\sqrt{\gamma})$ times less than the circulation-periodic velocities.

The complete flow is an extremely complex pattern of a set of Poiseuille confined flow on the velocity profile of which a "traveling" wave is superimposed from the wall with a periodic perturbation of the profile of the wall in the form of "oscillating" circulations, and superimposed on this there are stationary circulation-periodic flows etc. Further expansion of the series in l (or with respect to the parameter $ka \ll 1$) would give an additional set of hf "oscillating" circulations, but their intensity is obviously small.

For the lf limiting case with $\gamma \gg 1$, $\beta \gg 1$, $\alpha_1 \ll 1$ the final solution has the form

$$u_1 = 4e^{-1}\frac{\text{ch}\,1e^{y}(y+1)+(1-e\,\text{sh}\,1)e^{-y}(y-1)}{\text{sh}^2 1}\sin x\cos t;$$

$$v_1 = 2\frac{(1-e\,\text{sh}\,1)e^{y}y^{-1}-e^{-1}\text{ch}\,1e^{y}(y+1)}{\text{sh}^2 1}\cos x\cos t;$$

$$p_1 = 8\frac{\gamma}{\beta}\frac{(1-e\,\text{sh}\,1)e^{-y}(y-1)-e^{-1}\,\text{ch}\,1\,e^{y}(y+1)}{\text{sh}^2 1}\cos x\cos t.$$

The qualitative nature of the flow pattern is of the same form as for the case of hf oscillations. However, the nonstationary effects are less pronounced here. On the convexities and waviness ($\sin x = 0.1$ and $\varepsilon = \pm\varepsilon$) the flow is laminar. In the zeroth approximation the Poiseuille velocity profile is corrected by the motion of the walls with a velocity distribution law which is practically linear with depth, on which are superimposed the effect of the first approximation, i.e., the waviness of the wall manifests itself.

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